Transfer function method for measuring characteristic impedance and propagation constant of porous materials

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A method for measuring the characteristic impedance and propagation constant of porous materials is described in this paper. Measurements were performed based on a surface impedance method that required a set of distinct acoustic impedances derived at the material surface. This requirement is satisfied by arbitrarily changing the air space depth behind the material, and then a new formulation is derived so that a recently developed method of determination, called the transfer function method, can be applied. An appropriate set of air space depths is also discussed. Glass wool and porous aluminum were used to assess the usefulness of the present method. The normal acoustic impedance and normal absorption coefficient of the test materials with arbitrary thicknesses or with an arbitrary air space depth behind them were calculated from the obtained characteristic impedance and propagation constant and were compared with the measured values that were obtained directly by using the transfer function method. The good agreement achieved suggests that the present method is reliable and effective enough to measure the characteristic impedance and propagation constant over a broadband frequency range.

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INTRODUCTION

Porous materials, such as glass wool or foam, are generally used to attenuate noise. In addition to these well-known materials, new materials such as porous metal, Kevlar, or ceramic have been developed to satisfy the need for a porous material that can be used at high temperatures, can be exposed to high-speed air flow, or whose physical characteristics will remain unchanged when exposed to a chemical gas atmosphere.

The most fundamental acoustic property of these porous materials is their sound absorption coefficient. This may be measured by using an acoustic impedance tube. Several papers[1–3] have reported using a relatively recently developed transfer function method rather than the conventional standing wave ratio method to measure the sound absorption coefficient. This new method uses a broadband random signal as a sound source, so that the normal sound absorption coefficient and normal acoustic impedance are measured in a short time over a broadband frequency range. Though the sound absorption coefficient and acoustic impedance are measured more efficiently by this new method, to obtain them is not sufficient in order to use the porous material effectively. Further, it is more important to obtain the characteristic impedance and propagation constant of the porous material because the material might essentially be characterized by these properties. For example, previous work[4] has shown that the porous material itself must be considered as a medium in which the sound wave transmits, assuming the surface of the porous material is not locally reacting.[5] In that case, the porous material can be characterized not by the normal acoustic impedance, but by the characteristic impedance and the propagation constant. Even if the surface is assumed to be locally reacting, these properties are useful because the sound absorption coefficient and the acoustic impedance can be calculated from them.

As mentioned above, it is important to obtain both the characteristic impedance and propagation constant. Two approaches are used to obtain these properties. One is to establish an appropriate description of the porous material from the first principle: the propagation of the sound through it. The characteristics of sound propagation through porous materials have been investigated by several researchers,[6–11] but most notably by Zwikker and Kosten.[6] To utilize the theory in Ref. 6, it is necessary to obtain certain parameters, such as flow resistance and structure factor, that are measured by means of nonacoustical experimentation.

Another approach involves measuring these properties directly using an acoustical experiment. The most straightforward method was developed by Scott.[12] His technique is based on measuring the attenuation and phase change of an incident sound wave inside the porous material. The material was thick enough to prevent the wave from reflecting in the impedance tube.

The surface impedance measurement method was studied by Yaniv.[13] The characteristic impedance and propagation constant were calculated from a set of distinct acoustic impedances derived by measurements taken at the surface of the porous material. The distinct impedances were achieved by changing the air space depth behind the porous material:

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(1) porous material backed by a rigid wall and (2) porous material backed by one-quarter wavelength air space terminated in the rigid wall.

Smith and Parrott[14] achieved the distinct impedances by using two different thicknesses of material backed by a rigid wall. They called their method the "two-thickness method" and referred to Yaniv's method as the "two-cavity method." They measured acoustic impedances by using the conventional standing wave ratio method.

Terao[15] used the two-thickness method, where acoustic impedances were measured using the transfer function method.

The work reported here is an attempt to improve the two-cavity method, which was initially described in Ref. 6 and was studied further by Yaniv.[13] Previously, the two-cavity method required the acoustic impedances behind the material to be zero and infinity. The latter is easily achieved by a rigid wall. The former is achieved at each frequency by changing the air space depth to one-quarter wavelength, terminated by a rigid wall. This means that the air space depth must be changed at every frequency of interest.

In this paper, a new formulation is derived to calculate the characteristic impedance and propagation constant when the porous material is backed by an arbitrary impedance. Considering the acoustic impedance behind the material as a closed-tube impedance that can be obtained theoretically, it becomes unnecessary to change the air space depth at every frequency according to the new formulation. This means that the transfer function method can also be applied to the two-cavity method.

As compared with the two-thickness method, the improved two-cavity method has the advantage of reducing the number of times of mounting test materials, as well as elimination of the variation associated with test-piece mounting and the variation of the material itself between two test pieces.

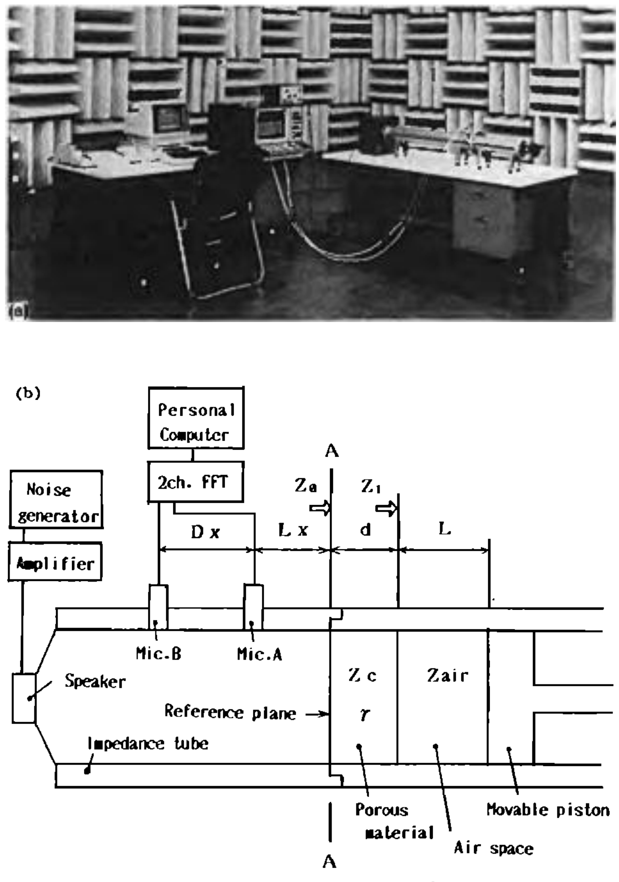

FIG. 1. (a) Experimental apparatus used to measure the characteristic impedance and propagation constant; (b) block diagram of the impedance tube and accessory equipment.

I. FORMULATION FOR A NEW TWO-CAVITY METHOD

A layer of homogeneous porous material is considered, as shown in Fig. 1(b). The acoustic impedance Z_0 at a reference surface can be related to the characteristic impedance Z_c, the propagation constant γ, the acoustic impedance behind the porous material Z_1, and the material thickness d, as follows[6]:

$$Z_0 = Z_c \frac{Z_1 \cosh(\gamma d) + Z_c \sinh(\gamma d)}{Z_1 \sinh(\gamma d) + Z_c \cosh(\gamma d)}. \tag{1}$$

Then, Eq. (1) yields

$$\frac{Z_0 + Z_c}{Z_0 - Z_c}\frac{Z_1 - Z_c}{Z_1 + Z_c} = \exp(2j\gamma d). \tag{2}$$

The right side of Eq. (2) is a function of d and γ. While the thickness remains constant, the same equation is obtained even if Z_0 and Z_1 are changed to other values. Let another be represented by ($'$); then Z_0 and Z_1 are replaced by Z_0' and Z_1', respectively. We obtain

$$\frac{Z_0 + Z_c}{Z_0 - Z_c}\frac{Z_1 - Z_c}{Z_1 + Z_c} = \frac{Z_0' + Z_c}{Z_0' - Z_c}\frac{Z_1' - Z_c}{Z_1' + Z_c}. \tag{3}$$

Solving Eqs. (2) and (3) for Z_c and γ gives

$$Z_c = \pm\left(\frac{Z_0 Z_0'(Z_1 - Z_1') - Z_1 Z_1'(Z_0 - Z_0')}{(Z_1 - Z_1') - (Z_0 - Z_0')}\right)^{1/2}, \tag{4}$$

$$\gamma = \frac{1}{2jd}\ln\left(\frac{Z_0 + Z_c}{Z_0 - Z_c}\frac{Z_1 - Z_c}{Z_1 + Z_c}\right), \tag{5}$$

where the sign in Eq. (4) is selected so as to let the real part of Z_c be positive. Using Eqs. (4) and (5), Z_c and γ can be calculated from the measured impedances Z_0 and Z_0' and the impedances of a closed tube with depth L and L', i.e.,

$$Z_1 = -jZ_{\text{air}} \cot(kL), \tag{6}$$

$$Z_1' = -jZ_{\text{air}} \cot(kL'), \tag{7}$$

where k is the wavenumber of air and Z_{air} is the characteristic impedance of air.

To summarize, Eqs. (4) and (5) are derived without assuming Z_1 to be zero and Z_1' to be infinity. Considering them as theoretical closed-tube impedances and measuring the reference surface acoustic impedance twice using the transfer function method permit the determination of the characteristic impedance and propagation constant over a broadband frequency range.

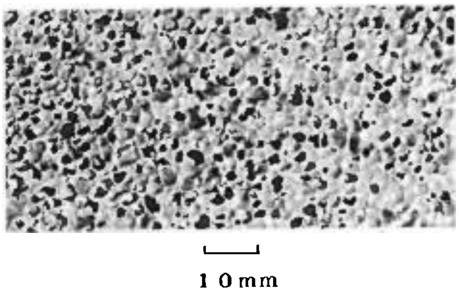

FIG. 2. Porous aluminum absorbing material.

II. EXPERIMENTAL PROCEDURE

Measuring the acoustic impedance using the random excitation technique is well documented in several papers.[1–3] In this paper, we describe the experimental apparatus utilized to mount the porous material to accomplish an arbitrary air space depth.

The experimental apparatus is shown in Fig. 1(a) and (b). The impedance tube, having an internal diameter of 87.5 mm and a length of 1 m, is terminated by a loudspeaker and a movable brass piston. The impedance tube can be separated at cross section *A–A*. The test material was cut into circular slices with flat faces and attached to the tube so that one reference plane was accurately established at cross section *A–A*, and another was "flush" against the surface of the movable piston. After visual inspection of the mounted test material, the tubes were connected. Then, the piston was moved backward to create the arbitrary air space depth. The air space depth L was measured by the scale attached to the tube. The distance Lx between the reference surface and microphone A was set at 17 mm, and the distance Dx between the two microphones was set at either 70 or 300 mm, according to the frequency range of interest.

A random signal was generated from the loudspeaker, and the transfer function H between the two microphones was measured using a two-channel fast Fourier transform. The acoustic impedance was obtained from

$$Z_0 = jZ_{\text{air}} \frac{-H\sin(kLx) + \sin[k(Lx+Dx)]}{H\cos(kLx) - \cos[k(Lx+Dx)]}. \quad (8)$$

After measuring Z_0 with air space depth L, the piston was moved to another depth L', and Z'_0 was measured using the same procedure.

Two different test materials were used for the experiment: one, a glass wool; and the other, a porous aluminum absorbing material (Fig. 2, hereafter referred to as a porous aluminum). The dimensions of the test materials and air space depths used are shown in Table I. The glass wool test material was 50 mm thick, while the porous aluminum test material was 20 mm thick. In order to calculate the characteristic impedance and propagation constant, it would be sufficient to measure the impedances based on two different air space depths, but in this paper the impedances with six different air space depths were measured individually to investigate whether the combination of air space depths affected the experimental accuracy when the present method was used.

III. EXPERIMENTAL RESULTS

The measured characteristic impedance of the glass wool is shown in Fig. 3(a). These results are normalized by the characteristic impedance of the air, Z_{air}. The solid lines represent the complex characteristic impedance. The experimental results were presented over a frequency range of from 300 Hz to 2 kHz, where the experimental error caused by the distance of separation between the microphones might be negligible. It is possible to measure the characteristic impedance over higher or lower frequency ranges by shortening or lengthening the distance between the microphones.

The complex propagation constant (rad/m) of the glass wool is shown in Fig. 3(b) over the same frequency range as the characteristic impedance. The measured data shown in Fig. 3(a) and (b) were obtained using distinct sets of air space depths: $(L, L') = (20\text{ mm}, 40\text{ mm})$, (20, 70), (20, 100), (40, 70), (40, 100), and (70, 100). From the graph, it can be seen that there is very good agreement among the six distinct measured values.

On the other hand, Fig. 4 shows the characteristic impedance of glass wool calculated from sets of air space depths different than those used for the data shown in Fig. 3(a). The air spaces used for the case shown in Fig. 4 were: $(L, L') = (20, 150)$, (20, 170), (40, 150), (40, 170), and (70, 170). In general, the figures show the same trend as was evident in Fig. 3(a), but significant systematic discrepancies appear in the 1100- to 1700-Hz frequency range. The reason for this will be discussed later. The results suggested that there is an ideal set of air space depths of which the characteristic impedance and propagation constant can be calculated correctly.

Figure 5(a) and (b) shows the characteristic impedance and propagation constant of the porous aluminum. Because porous aluminum has attractive acoustic characteristics at low frequencies, it was measured over a range of frequencies of from 100 to 450 Hz. This was accomplished by spacing the microphones 300 mm apart. The test material

TABLE I. Properties of the materials tested.

Sample	Density (kg/m³)	Diameter (mm)	Thickness d (mm)	Air space depth L (mm)
Glass wool	30	87.5	50	20, 40, 70, 100, 150, 170
Porous aluminum	250	87.5	20	20, 40, 70, 100, 150

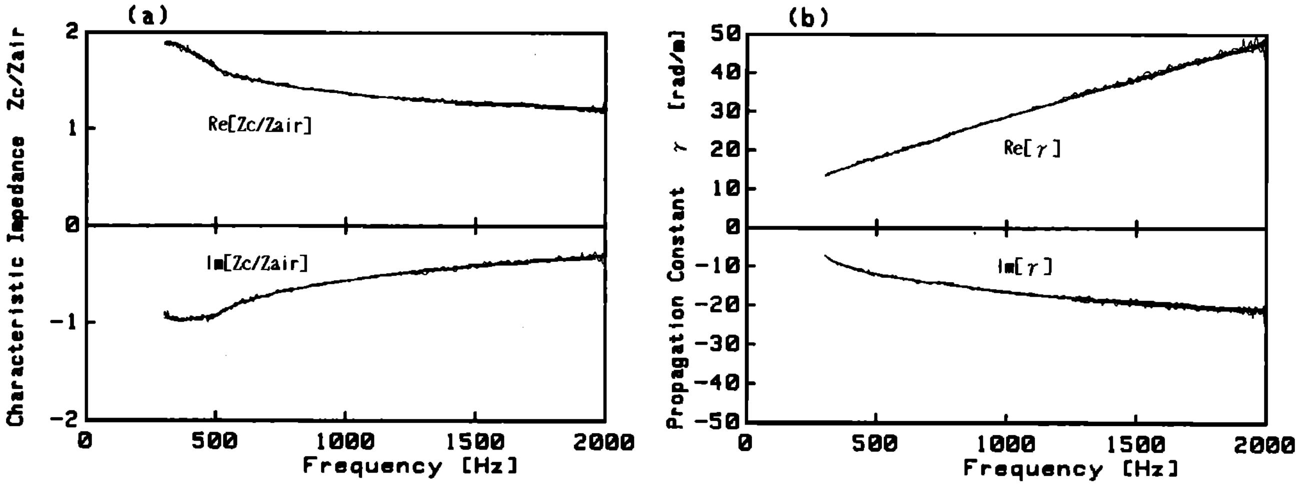

FIG. 3. (a) The characteristic impedance of glass wool normalized by air; (b) the propagation constant of glass wool, where both properties were obtained at $d = 50$ mm and $(L, L') = (20$ mm, 40 mm$)$, (20, 70), (20, 100), (40, 70), (40, 100), and (70, 100).

was 20 mm thick, and five sets of air space depths were used: $(L, L') = (20, 40)$, (20, 70), (20, 100), (40,70), and (40, 100). The characteristic impedance was normalized by that of air. It can be seen that there is good agreement among the measured values obtained from distinct sets of air space depths over a frequency range of from 150 to 400 Hz. While there are slight discrepancies below 150 Hz and above 400 Hz, a consistent trend is observed.

IV. DISCUSSIONS

The characteristic impedance and propagation constant were measured for glass wool, a traditional porous material, and for porous aluminum, a newly developed metallic absorbing material. In the above experimental results, it is shown that both properties can be measured when utilizing an appropriate set of air space depths. The discussion below describes two factors: (1) the selection of an appropriate set of air space depths and (2) the verification of measured acoustic properties.

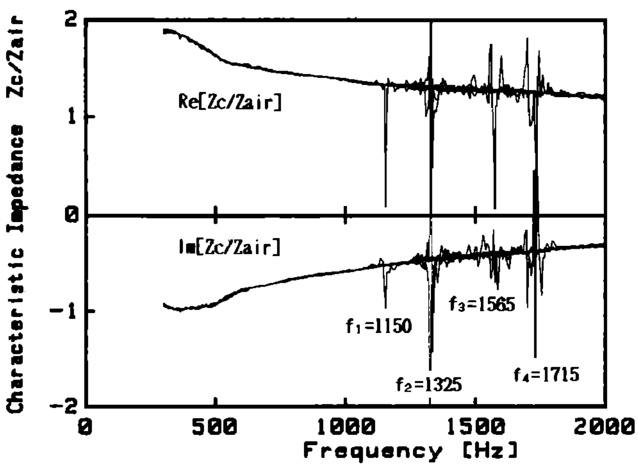

FIG. 4. The characteristic impedance of glass wool normalized by air, where it was obtained at $d = 50$ mm and $(L, L') = (20$ mm, 150 mm$)$, (20, 170), (40, 150), (40, 170), and (70, 170).

A. The selection of an appropriate set of air space depths

If Z_1 approaches Z_1' in Eq. (4), then Z_c approaches $\pm Z_1$; i.e., the characteristic impedance of the porous material is equal to the acoustic impedance of the closed tube. This incorrect conclusion is caused by elimination of the terms $(Z_0 - Z_0')$ in Eq. (4), and that situation arises when the air spaces L, L' and the frequency f satisfy the following equation:

$$f(L - L') = nc/2, \tag{9}$$

where c is the speed of sound in the air and $n = 1, 2, 3, \ldots$. The frequencies of the distinctive peaks, f_1–f_4, shown in Fig. 4, are in complete agreement with the frequencies calculated from Eq. (9), so the appropriate set of air space depths must be selected so as to not satisfy Eq. (9).

B. The verification of measured acoustic properties

The present investigation was initiated to improve the two-cavity method so as to calculate the characteristic impedance and propagation constant, even if the acoustic impedances Z_1 and Z_1' are not zero and infinity, respectively. The previous two-cavity method has been verified in studies by Yaniv and Smith and Parrott and compared with that obtained using Scott's method or the theoretical model due to Beranek. In this paper, a comparison is made between the calculated and measured normal acoustic impedance and absorption coefficient. Substituting the measured Z_c and γ into Eq. (1), the normal acoustic impedance can be calculated. A subsequent calculation is made to determine the absorption coefficient. Both of these quantities are also measured by using the transfer function method directly.

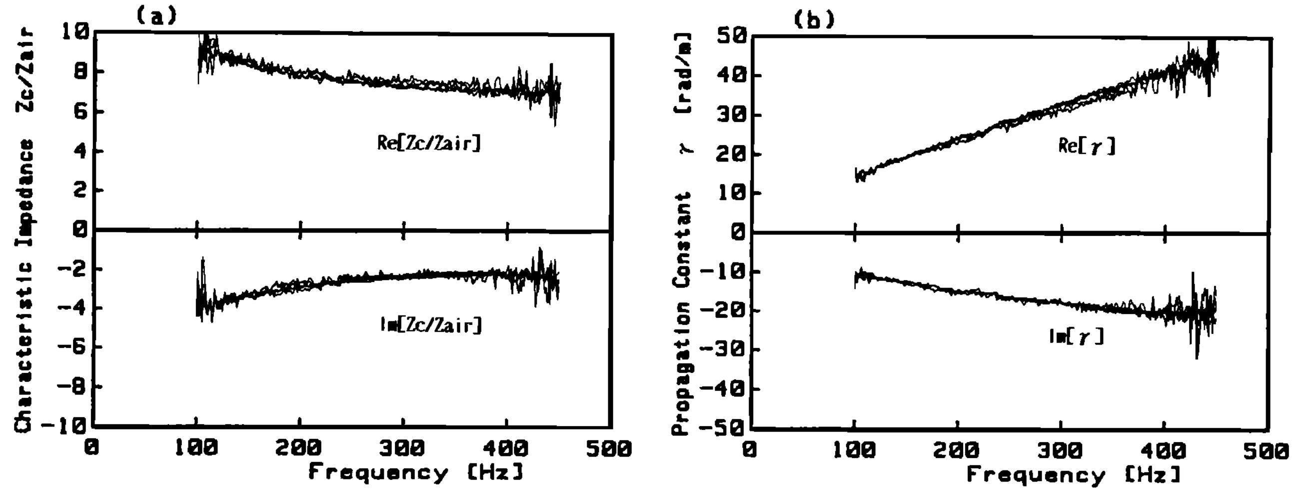

FIG. 5. (a) The characteristic impedance of porous aluminum normalized by air; (b) the propagation constant of porous aluminum, where both properties were obtained at $d = 20$ mm and $(L, L') = (20$ mm, 40 mm), (20, 70), (20, 100), (40, 70), and (40, 100).

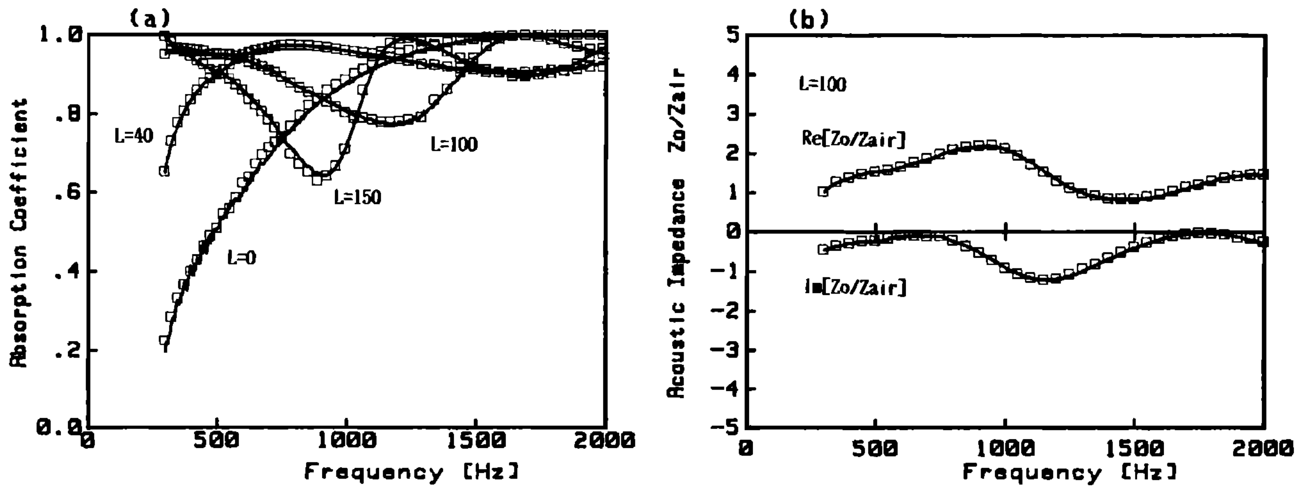

FIG. 6. (a) Comparison between calculated (—) and measured (□) normal absorption coefficient for 50-mm-thick glass wool with an air space depth of $L = 0$, 40, 100, and 150 mm; (b) comparison between calculated (—) and measured (□) normal acoustic impedance for 50-mm-thick glass wool with a 100-mm air space depth, where the calculation was based on the properties that were obtained at $d = 50$ mm and $(L, L') = (20$ mm, 70 mm).

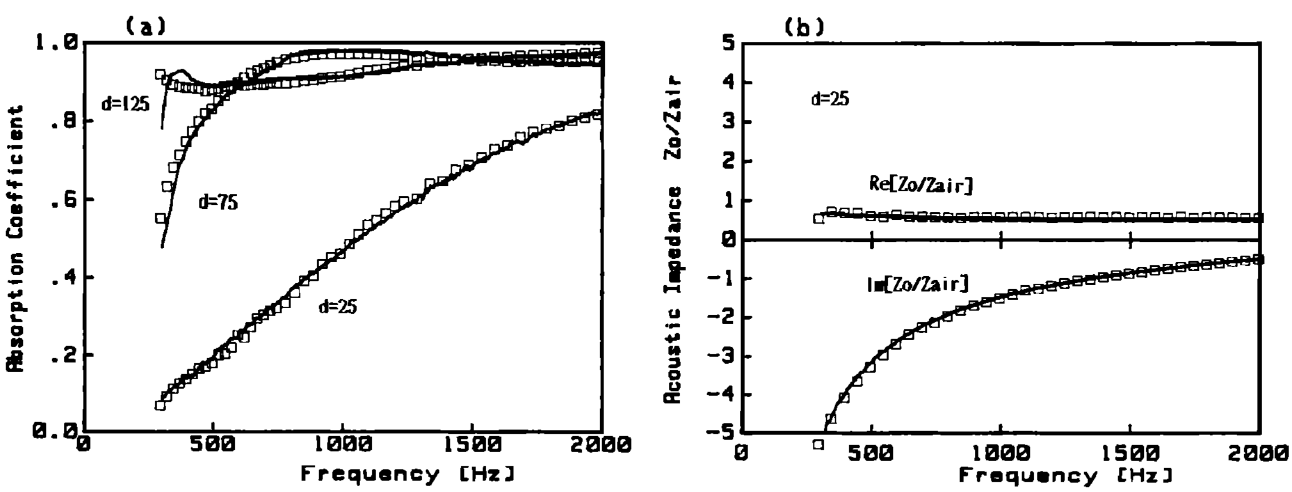

FIG. 7. (a) Comparison between calculated (—) and measured (□) normal absorption coefficient for 25-, 75-, and 125-mm-thick glass wool backed by a rigid wall; (b) comparison between calculated (—) and measured (□) normal acoustic impedance for 25-mm-thick glass wool backed by a rigid wall, where the calculation was based on the properties that were obtained at $d = 50$ mm and $(L, L') = (20$ mm, 70 mm).

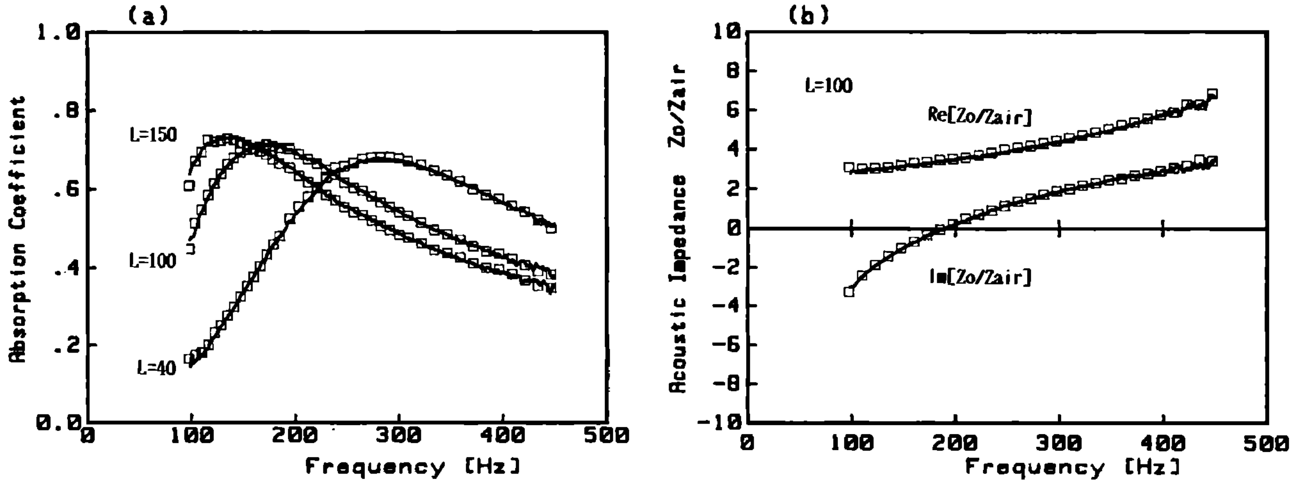

FIG. 8. (a) Comparison between calculated (—) and measured (□) normal absorption coefficient for 20-mm-thick porous aluminum with air space depths of $L = 40$, 100, and 150 mm; (b) comparison between calculated (—) and measured (□) normal acoustic impedance for 20-mm-thick porous aluminum with a 100-mm air space depth, where the calculation was based on the properties that were obtained at $d = 20$ mm and $(L, L') = (20$ mm, 70 mm).

Figure 6(a) shows the normal sound absorption coefficient for 50-mm-thick glass wool with an air space depth of $L = 0$, 40, 100, and 150 mm, respectively. In the graph, the open square represents the measured values derived by using the transfer function method, while the solid line represents the calculated values derived from measured values of Z_c and γ that were obtained at $d = 50$ mm and $(L, L') = (20$ mm, 70 mm). The normal acoustic impedance is shown in Fig. 6(b) for 50-mm-thick glass wool with an air space depth of 100 mm. The measured acoustic impedance is represented by the open square, while the calculated acoustic impedance is represented by the solid line. The real and imaginary parts of the acoustic impedances are normalized by the characteristic impedance of the air. As may be seen, the calculated and measured data show excellent agreement.

In Fig. 7(a) and (b), the normal sound absorption coefficient for 25-, 75-, and 125-mm-thick glass wool test materials backed by a rigid wall and the normal acoustic impedance for a 25-mm-thick glass wool backed by the rigid wall is shown. From these graphs, it can be seen that the calculated values are in good agreement with the measured values when the thickness of the test material is a variable.

Using the characteristic impedance and propagation constant obtained at $d = 20$ mm and $(L, L') = (20$ mm, 70 mm) in Fig. 5, the normal absorption coefficient and the normal acoustic impedance were calculated for 20-mm-thick porous aluminum with an air space depth of $L = 40$, 100, and 150 mm. These data and corresponding measured data obtained using the transfer function method directly are shown in Fig. 8(a) and (b). The calculated and measured normal absorption coefficient and normal acoustic impedance are also shown in Fig. 9(a) and (b) for 10-, 30-, and

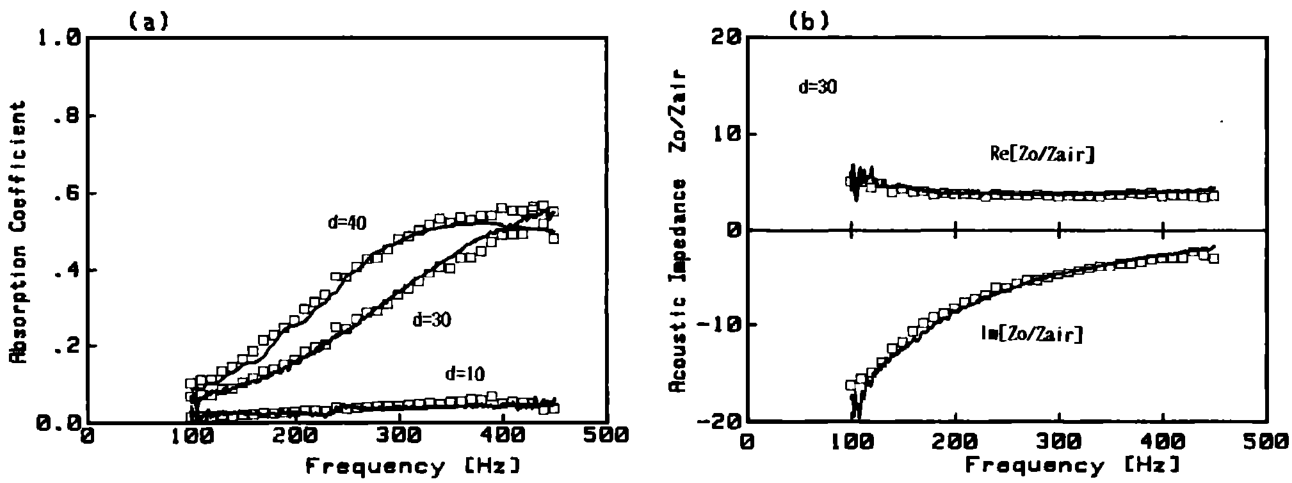

FIG. 9. (a) Comparison between calculated (—) and measured (□) normal absorption coefficient for 10-, 30-, and 40-mm-thick porous aluminum test materials backed by a rigid wall; (b) comparison between calculated (—) and measured (□) normal acoustic impedance for 30-mm-thick porous aluminum backed by a rigid wall, where the calculation was based on the properties that were obtained at $d = 20$ mm and $(L, L') = (20$ mm, 70 mm).

40-mm-thick porous aluminum test materials backed by a rigid wall. It can be seen that the calculated values for the porous aluminum are in good agreement with the measured ones.

From these results, it can be concluded that the sound absorption characteristics of a porous material can be determined accurately from the measured characteristic impedance and the propagation constant. It also indicates that the sound absorption characteristics of a porous material can be predicted easily using the present method.

V. SUMMARY

In this paper, an improved two-cavity method to measure the characteristic impedance and propagation constant of porous materials was applied experimentally to two different porous materials. The characteristic impedance and propagation constant can be measured easily over a broad frequency range by using the transfer function method. The selection of an appropriate set of air space depths is discussed, and it is shown that the sound absorption coefficient and acoustic impedance of porous materials with arbitrary thickness or with an arbitrary air space depth may be predicted accurately using the measured characteristic impedance and propagation constant. This result indicates that the present method is useful in predicting the sound absorbing capability of acoustical materials in practical applications.

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